Network Models in crypto asset management

Paolo Giudici, Iman Abu Hashish, Branka Hadji Misheva

Fintech Laboratory and PhD
Department of Economics and of Computer Engineering
University of Pavia
E-mail: giudici@unipv.it; Facebook: Fintech Network

Topics

- Topic 1: Determinants of BitCoin exchange prices - A network VAR approach
- Topic 2: Conditional independence relations between BitCoin traders.

Crypto Assets

- A cryptocurrency can be defined as "a digital asset designed to work as a medium of exchange using cryptography to secure the transactions and to control the creation of additional units of the currency";
- They are considered one of the largest markets in the world which remain to be unregulated.
- Within the last decade, these digital currencies operating independently of a central bank, have massively grown in popularity and consequently in price.
- Looking at the Bitcoin alone, the market capitalization of this cryptocurrency came close to $330 billion USD in December 2017 (CoinMarketCap, 2018).

Topic 1: our contribution

- Estimate the correlation structure among bitcoin exchange prices in different market exchanges.
- To understand whether prices are determined endogeneously or whether they are affected by "classical" assets.
- Investigate whether correlation network models can improve price predictive accuracy.

Data - crypto exchange markets

Table: Exchange markets by daily trading volumes (cryptocoincharts.info)

Exchange	Market share
Bitfinex	42%
Coinbase	6%
Bitstamp	5%
Hitbtc	3%
Gemini	2%
itBit	1%
Kraken	0.5%
Bittrex	0.5%

Data - crypto prices

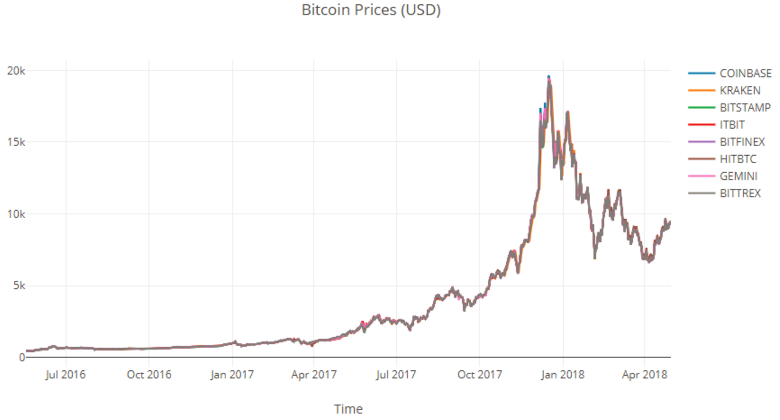
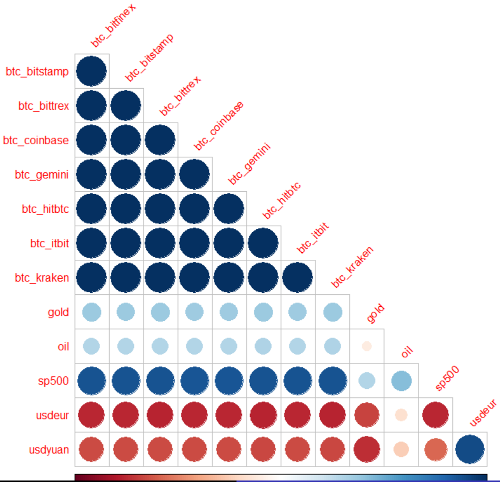


Table: Summary statistics

Price	Mean	St. Dev.	Min	Max
Bitfinex Bitcoin	3899.56	4274.46	435.61	19187.12
Coinbase Bitcoin	3919.05	4318.98	438.38	19650.01
Bitstamp Bitcoin	3899.04	4286.02	439.62	19187.78
HitBtc Bitcoin	3916.19	4297.17	436.36	19095.30
Gemini Bitcoin	3910.38	4306.36	437.57	19475.90
ItBit Bitcoin	3907.13	4300.32	438.61	19357.97
Kraken Bitcoin	3890.18	4272.55	433.50	19356.91
Bittrex Bitcoin	3893.83	4269.89	421.11	19261.10
Gold	1275.57	52.34	1128.42	1366.38
Oil	48.67	3.16	39.51	54.45
SP500	2414.78	212.308	2000.54	2872.87
USDYuan	6.67	0.19	6.26	6.96
USDEur	0.88	0.04	0.80	0.96

Correlation structure



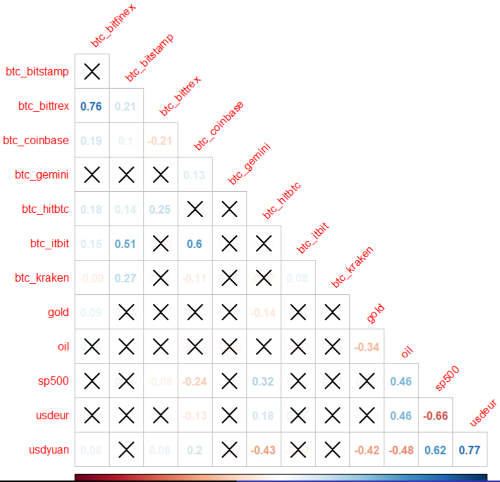
Partial Correlations

- From the inverse of the covariance matrix (A^{-1}, elements σ^{mn}), partial correlations can be derived:

$$\rho_{mn|rest} = \frac{-\sigma^{mn}}{\sqrt{\sigma^{mm}\sigma^{nn}}}.$$

- They represent correlations between two prices, conditional on the remaining prices of the system (*rest*).

Partial correlation structure



Partial Correlation network

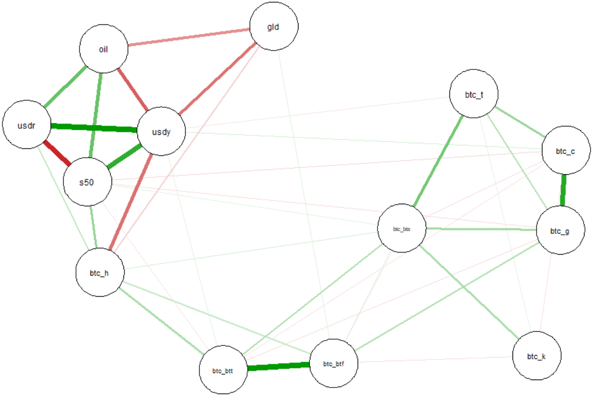


Figure: Partial correlation network model between prices.

Correlation network models - I

Let i be a market price (crypto, forex, share). We assume prices y_t^i follow a structural VAR process:

$$y_t^i = \sum_{p=1}^{T} \beta_p^i y_{t-p}^i + \sum_{j \neq i} \gamma^j y_t^j + \varepsilon_t^i.$$

or

$$Y_t = \sum_{p=1}^{T} A_p Y_{t-p} + B_0 Y_t + u_t,$$

where B_0 has null diagonal elements.

Correlation network models - II

We can transform the VAR to its reduced form:

$$Y_t = \Gamma_1 Y_{t-1} + ... + \Gamma_p Y_{t-p} + u'_t,$$

where

$$\begin{cases} \Gamma_1 = (\mathbb{I} - B_0)^{-1} A_1, \\ ... \\ \Gamma_p = (\mathbb{I} - B_0)^{-1} A_p, \\ u'_t = (\mathbb{I} - B_0)^{-1} u_t. \end{cases}$$

Need to estimate B_0 to derive $A_1, ..., A_p$. Note that $u'_t = B_0 u'_t + u_t$. Thus:

$$\begin{cases} [u'_t]^i = \sum_{j \neq i} b_0^j [u'_t]^j + [u_t]^i, \\ \gamma^{ij} = \sqrt{b_0^j b_0^i} = corr(y_i, y_j | rest) \end{cases}$$

Results: estimation

Table: Estimation components

Asset	Contemp	Autoreg
Bitfinex Bitcoin	452.31	0.74
Coinbase Bitcoin	453.55	0.68
Bitstamp Bitcoin	450.35	0.19
HitBtc Bitcoin	451.98	0.37
Gemini Bitcoin	450.01	0.20
ItBit Bitcoin	451.01	0.19
Kraken Bitcoin	451.02	0.28
Bittrex Bitcoin	452.15	0.69
Oil	4.03	0.95
SP500	197.26	0.97
USDYuan	0.51	0.99
USDEur	0.06	0.96
Gold	97.51	0.99

Results: predictive performance

Table: Predictive performance summary

Prices	RMSE full	RMSE autoreg
Bitfinex Bitcoin	267.37	293.49
Coinbase Bitcoin	550.10	579.98
Bitstamp Bitcoin	379.71	397.25
HitBtc Bitcoin	290.63	342.50
Gemini Bitcoin	792.22	786.58
ItBit Bitcoin	331.62	455.45
Kraken Bitcoin	718.51	676.35
Bittrex Bitcoin	288.84	305.51
Oil	0.55	0.58
SP500	5.90	6.35
USDYuan	0.02	0.02
USDEur	0.002	0.003
Gold	6.74	7.19

Conclusions

- We have proposed a correlation network model to understand price dynamics in bitcoin exchange markets
- The model is effective, from a descriptive viewpoint, and provides accurate predictions
- Crypto market prices are mostly correlated to each other and (still) diversified with other assets

Topic 2 - Background

- On 12th November 2017, someone moved 25,000 BitCoins, worth USD 159 million at the time, to an exchange.
- Bloomberg estimates that 40% of BitCoins are owned by approximately 1000 users. Any movement by these users can ripple through the market and cause major disruptions in the price of the cryptocurrencies.
- Having this in mind, the structure of the BitCoin network and the centrality of individual nodes within it, becomes crucial for understanding price and volume movements.

Our contribution

- To identify and describe the key drivers influencing BitCoin transaction volumes.
- Specifically, our research has two objectives:
  - to identify the network structure that emerges based on the BitCoin transaction volumes across different regions;
  - to model conditional independence relations between BitCoin traders in different regions thus identifying leader-follower links.

Data - I

- The data used covers Bitcoin transactions in the time period 25.02.2012 to 17.07.2017.
- Transaction volumes have been normalized with GARCH (1,1) model.
- Transactions are grouped both by region and size of transactions resulting in 60 groups.

Data - II

- 1-10: Transactions from Africa, group 1 are the 10% lowest transactions, group 10 are the 10% highest;
- 11-20: Transactions from Asia, group 11 are the 10% lowest transactions, group 20 are the 10% highest;
- 21-30: Transactions from Europe, group 21 are the 10% lowest transactions, group 30 are the 10% highest;
- 31-40: Transactions from North America, group 31 are the 10% lowest transactions, group 40 are the 10% highest;
- 41-50: Transactions from Oceania, group 41 are the 10% lowest transactions, group 50 are the 10% highest;
- 51-60: Transactions from South America, group 51 are the 10% lowest transactions, group 60 are the 10% highest;

Methodology

- Graphical models have been extensively used as a general framework for modelling conditional independence relations between variables;
- In this study, we implement several models for the purpose of obtaining the true structure of the network:
    - partial correlation models;
    - graphical LASSO;
    - Granger-causality.

Partial correlation networks

Let $\sum$ denote a variance-covariance matrix.

$$y \sim N(O, \Sigma)$$

$$K = \Sigma^{-1}$$

$$Cor(y_i, y_j | y_{-(i,j)}) = -\frac{K_{ij}}{\sqrt{K_{ii}\sqrt{K_{jj}}}}$$

Graphical LASSO

- One of the most popular methods for limiting the number of spurious links is the LASSO.
- It is based on the penalized likelihood for the precision matrix:

$$l(\Theta) = log\ \ det\Theta - tr(S\Theta) - \lambda_p \sum(|\Theta_{i,j}|)$$

- Where Θ is the precision matrix, S is the covariance matrix and λ_p is the penalty function

Granger Causality Networks

- In the Granger-sense x is a cause of y if it is useful in forecasting y. In this framework "useful" means that x is able to increase the accuracy of the prediction of y with respect to a forecast, considering only past values of y.
- The Granger Causality test leads to the construction of a directed graph $G = (V, E)$ where a set of vertices are connected to each other based of the significance of the causality test. Ex. if i Cranger causes j the network will contain a directed link from i to j.

Partial Correlation Network

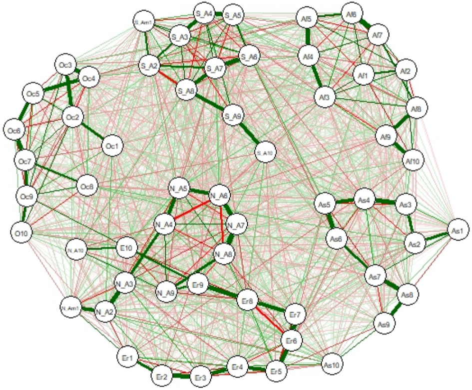


Figure: Unregularized partial correlation network

Partial Correlation Network (Testing for Significance, 1%)

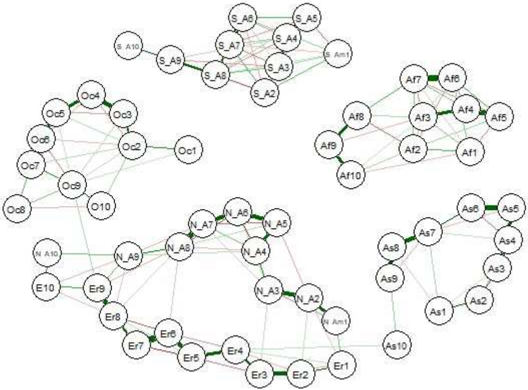


Figure: Partial correlation network (without edges that are not significant; alpha = 1%)

GLASSO

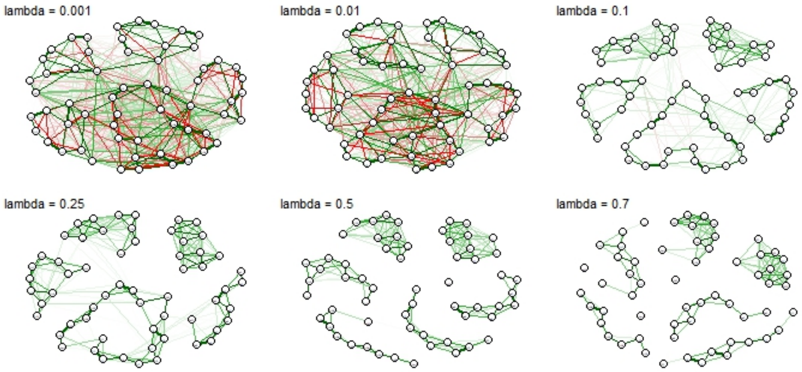


Figure: Networks based on GLASSO estimation [varying lambda]

Granger Causality Networks - I

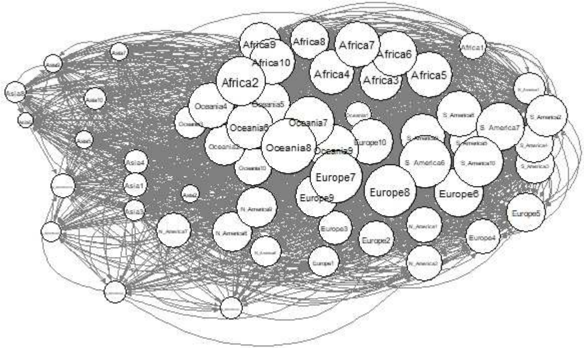


Figure: Granger Causality Graph (node size indicated OUT degree centrality)

Granger Causality Networks - II

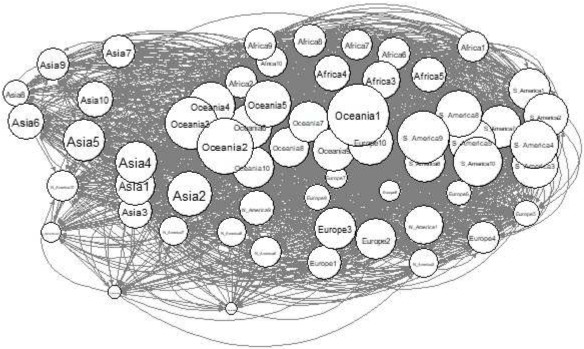


Figure: Granger Causality Graph (node size indicated IN degree centrality)